# 3D Point Cloud Registration Using Particle Swarm Optimization Based on Different Descriptors 

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#### Abstract

3D laser scanners generate points cloud for real objects in multiple scenes. Thus the registration is needed to put each scene in its own right position relative to the other scenes. The challenge of the registration process is to find the transformation matrix that realizes the accurate alignment at minimum computation time. This paper proposes a registration algorithm based on an objective function of Hausdorff distance. The objective function uses three descriptors as its own parameter individually. These descriptors are points coordinates, heat Kernel Signature (HKS) and Gaussian curvature ( $\kappa_{g}$ ). The minimization of objective function (Hausdorff distance) is attained using particle swarm optimization (PSO) method. The comparison showed the algorithm based on distance is the best results.


Keywords: 3D point cloud, Particle Swarm Optimization (PSO), distance, Heat Kernel Signature, Gaussian curvature, 3D data laser scanner.

## 1 Introduction

Advanced technologies in Laser scanning have become vital tools in many engineering applications. It can be captured large amount of data within high accuracy and short time. Laser scanners take physical object's same size and shape into the computer world as a digital three dimensional representation. These scanners generate the group of points having particular properties such as ( $\mathrm{x}, \mathrm{y}, \mathrm{z}$, Intensity, RGB) which are called point cloud. Moreover, laser scanning technologies are used in many applications such as industrial, archaeology, historical studies, 3D object scanning, 3D mapping, 3D localization and cultural heritage [1]. The scanning process of any object (or site) is carried out from different positions around the object because of occlusion. Thus, the scanning process delivers number of scans corresponding positions. In order to construct the digital object (or site) these scans need to register. The registration process is performed by finding the optimal values of transformation matrix [2] that realizes the possible good alignment between the all scans. The alignment between two scans can be accomplished according certain objective function and the methods of its solution. Several efforts have been made to find methods solving the registration point cloud problem. Some of these methods depend on directly point cloud or photometric properties such as Random sample consensus (RANSAC), Iterative Closest Point (ICP) and Scale-invariant feature transform (SIFT). RANSAC[3] algorithm was developed to deal with the data that contains outliers and improve the estimation of the rigid transformation. (ICP) [4] algorithm was utilized to get the best rotation and translation that minimizes an error metric based on the distance to align two scans. Abbas et al. [5] offered a registration method which utilizes the SIFT algorithm work on distinctive features which are extracted from both aerial images and LiDAR intensity data.

Gruen and Akca[6] used Generalized Gauss-Markoff for estimation the transformation parameters of 3D scenes. Myronenko and Song [7] presented an alignment technique for rigid, affine and non-rigid transformation cases, called Coherent Point Drift. Potmesil [8] calculated the distance between points to normal plane to maximize evaluation of registration. Zhang [9] proposed a thresholding technique that used a robust statistical method based on the distance distribution to limit the maximum distance between scans. Glomb [10] used feature detection method for registration, in which extended Harries descriptor to 3D shapes. Dina et al. [11] applied combined 3D Sobel-Harris operator on point cloud to detect the interest points. Another class of methods depend on feature which represent the shape as a collection of some local feature descriptors. The advantage of this descriptor is invariant to translation and rotation. This descriptor captured geometric arrangement of points, surfaces and objects for solving registration problem. So authors depend on descriptors such as Spin image [12], heat kernel signature (HKS) [13]and curvature [14]. Spin image [12] descriptor object properties with the robustness to partial views and clutter of local shape descriptions. Marco et al. [15] using spin image for partial views registration. HKS describes the intrinsic local shape based on diffusion scale -space analysis. Jian Sun et al. [13] present a new technique to extract all the information contained in the heat kernel and characterizes the shape up to isometry called heat kernel signature (HKS). Bayumy [16] used Heat Kernel Signature to register scenes at different ranges and optimize solution by an Artificial Bee colony optimization. Jing Huang and Suya You [17] presented some descriptor for 3D self-similarity registration such as normal, curvature and photometry and also combined both geometric and photometric information. The solution of above method is iterative methods.

This method is time-consuming, so heuristic method used for reaching solution quickly. Automatic registration process using heuristic need a robust global optimizer. Optimization methods are categorized as derivative and derivative free methods. The popular examples of derivative methods are gradient such as Newton's Methods[18]. Pottmann et al.[18] register scene by minimizing squared distances to the tangent planes and optimizing using a Gauss-Newton iteration. But these methods need good initial values for estimating in order to avoid the local minimum since the transformation parameters are generally nonconvex and irregular. Problem of derivative method can get local minimum. The solution of this problem is a derivative free methods such as using Genetic Algorithm [19] and simulated annealing[20]. The genetic algorithm (GA), which is one of the global optimization techniques, has been proposed for point cloud registrations. Silva et al.[21]proposed a combination between genetic algorithm and hill-climbing to solve range image registration problem. Disadvantage of genetic algorithm takes a larger computation time and lacks in fine tuning capabilities. Bilbor and Snyder[22] apply simulated annealing(SA) to register 3D point cloud and reduce the computation by modified iteratively during the minimization. This modified is built $k$-tree around minimizers of the cost function. Recently, a global optimization technique called particle swarm optimization has been proposed, which is a stochastic, population-based evolutionary computer algorithm.

The rest of this paper is organized as follows: we introduce in section 2 proposed registration methodology. In section 3 particle swarm algorithm (PSO) is introduced. In section 4 experimental setup. In section 5 shows Experimental Result and section 6 conclusions.

## 2 Proposed Registration Methodology

The registration can be accomplished by minimizing objective function which relies on the idea of Hausdorff distance[23]. Consider, it are Given a model point (moving scene) set $\mathrm{M}=\left\{m_{1}, m_{2}, \ldots \ldots . m_{n}\right\}$ and a data point (source scene) set $\mathrm{S}=\left\{s_{1}, s_{2}, \ldots \ldots . s_{g}\right\}$. The registration's process is achieved by applying the transformation matrix (translations $\boldsymbol{t}$ and rotations $R$ ), which changes the position the model points (M) relative the data points (S). Hence Hausdorff distance is evaluated according to that position. The minimization of objective function (Hausdorff distance) is attained using particle swarm optimization (PSO) method.

Applying the transformation matrix on M's set yields to:

$$
\vec{\chi}_{M}^{T}=\left[\begin{array}{ccc}
1 & 0 & 0  \tag{1}\\
0 & \cos \theta & \sin \theta \\
0 & -\sin \theta & \cos \theta
\end{array}\right]\left[\begin{array}{ccc}
\cos \alpha & 0 & \sin \alpha \\
0 & 1 & 0 \\
-\sin \alpha & 0 & \cos \alpha
\end{array}\right]\left[\begin{array}{ccc}
\cos \beta & \sin \beta & 0 \\
-\sin \beta & \cos \beta & 0 \\
0 & 0 & 1
\end{array}\right] \vec{\chi}_{M}+\left[\begin{array}{l}
t_{x} \\
t_{y} \\
t_{z}
\end{array}\right]
$$

Where $\vec{\chi}$ is Cartesian coordinate $\mathrm{x}, \mathrm{y}, \mathrm{z},(\theta, \alpha, \beta)$ are the rotation angles around axes $\mathrm{x}, \mathrm{y}, \mathrm{z}$ respectively, $\left(t_{x}, t_{y}, t_{z}\right)$ are the translation along axes $\mathrm{x}, \mathrm{y}, \mathrm{z}$ respectively and T is transformed set.

Objective function (f) (Hausdorff distance (h)) between the two sets ( $\mathrm{M}^{T}, \mathrm{~S}$ ) yields to:

$$
\begin{gather*}
f=\min \left(h\left(S, M^{T}\right)\right)  \tag{2}\\
h\left(S, M^{T}\right)=\max _{S}\left(\min _{M^{T}}\left(d\left(s, m^{T}\right)\right)\right) \tag{3}
\end{gather*}
$$

Our algorithm preforms the registration using three descriptors individually. These descriptors are points coordinates, heat Kernel Signature (HKS) and Gaussian curvature ( $\kappa_{g}$ ).
Then, The objective function yields to:
A. Points coordinates $\quad: h\left(S, M^{T}\right)=\max _{S}\left(\min _{M^{T}}\left(d\left(\vec{\chi}_{S}, \vec{\chi}_{M}^{T}\right)\right)\right)$
B. Heat Kernel Signature $: h\left(S, M^{T}\right)=\max _{S}\left(\min _{M^{T}}\left(d\left(H K S_{S}, H K S_{M}^{T}\right)\right)\right)$
C. Gaussian curvature $: h\left(S, M^{T}\right)=\max _{S}\left(\min _{M^{T}}\left(d\left(\kappa_{S}, \kappa_{M}^{T}\right)\right)\right)$

## 3 Heat Kernel Signature (HKS) Descriptor

HKS was introduced in 2009 by Jian Sun et al. [13] . It is based on heat kernel, which is a fundamental solution to the heat equation. HKS introduced shape descriptors which are based on the Laplace-Beltrami operator associated with the shape[24].
Some fact about heat diffusion on Riemannian manifolds present to introduce heat kernel. The heat diffusion process given by the heat equation

$$
\begin{equation*}
\left(\Delta+\frac{\partial}{\partial \tau}\right) u(x, \tau)=0 \tag{7}
\end{equation*}
$$

Where $\Delta$ denotes the positive semi-definite Laplace Beltrami operator[24], $u(x, \tau)$ describes the amount of heat on the surface at a point x and time $\tau$, where x is the spatial domain, and $\tau$ domain. In case of initial conditions $u(x, 0)=u_{0}(x)=\delta(x-z)$ it is called the heat kernel and denoted by $\mathrm{K}(\mathrm{x}, \mathrm{z})$ given by

$$
\begin{equation*}
K(x, z)=\sum_{i=0}^{\infty} e^{-\lambda_{i} \tau} \phi_{i}(x) \phi_{i}(z) \tag{8}
\end{equation*}
$$

Where $\lambda_{0}, \lambda_{1}, \lambda_{2}, \ldots \ldots \geq 0$ are eigenvalues and $\phi_{0}, \phi_{1}, \phi_{2}, \ldots \ldots$ are the corresponding eigenfunction of the LaplaceBeltrami operator, satisfying $\Delta \phi_{i}=\lambda_{i} \phi_{i}$.
Heat kernel signature shapes descriptor for register object which extract all the information contained in the heat kernel, and characterizes the shaping up to isometry. HKS is given by

$$
\begin{equation*}
h(x, \tau)=K(x, x)=\sum_{i=0}^{\infty} e^{-\lambda_{i} \tau} \phi_{i}^{2}(x) \tag{9}
\end{equation*}
$$

### 3.1 Gaussian curvature ( $\kappa_{g}$ )

The principal idea of curvature [25]at a point is to measure, how tangent line changes while moving to the point toward the neighbouring points. Let $p \in \mathbb{R}^{3}$ point, curvature $\kappa$ of curve $\overrightarrow{\mathrm{C}}$ passing through p defined as

$$
\begin{equation*}
\kappa=\frac{\left|\vec{C} * \vec{C}^{\prime \prime}\right|}{\left\|\vec{C}^{\prime}\right\|^{3 / 2}} \tag{10}
\end{equation*}
$$

The normal curvature $\kappa_{N}$ of curve $\vec{C}$ passing through the point P is defined by the following relation, known as

$$
\begin{equation*}
\kappa_{N}=\kappa \cos \alpha \tag{11}
\end{equation*}
$$

Where $\alpha$ is the angle between the curve's normal and the normal of surface $\vec{H}$.The minimum and maximum normal curvatures at $p$ is principal curvatures that can represent as $\kappa_{1}, \kappa_{2}$ of $\vec{H}$ at p . Gaussian curvature $\kappa_{g}$ is defined the product of the principal curvatures.

$$
\begin{equation*}
\kappa_{g}=\kappa_{1} * \kappa_{2} \tag{12}
\end{equation*}
$$

## 4 Particle Swarm Optimization

Eberhart and Kennedy[26] presented PSO based on affected by the social behaviour of animals such as swarms of birds and fish schooling. The population of PSO called particles, each particle analogous to a bird flying around in the multidimensional search space exploring for better regions. PSO particles are initialized randomly and then searches for optima by update generations. Update the velocity and the position of particles in each generation can be represented by:

$$
\begin{gather*}
v_{i}^{k+1}=v_{i}^{k}+\alpha_{1}\left[\gamma_{1, i}\left(\text { pbest }_{i}-x_{i}^{k}\right)\right]+\alpha_{2}\left[\gamma_{2, i}\left(\text { gbest }-x_{i}^{k}\right)\right]  \tag{13}\\
x_{i}^{k+1}=x_{i}^{k}+v_{i}^{k+1} \tag{14}
\end{gather*}
$$

The vectors $x_{i}^{k}$ and $v_{i}^{k}$ are the current position and velocity of the $i$ th particle in the $k$-th generation. Each particle has pbest ${ }_{i}$ is the best position of each individual and gbest is the global best position observed among all particles up to the current generation. The parameters $\gamma_{1,}, \gamma_{2} \in[0,1]$ are uniformly distributed random values and $\alpha_{1}, \alpha_{2}$ are acceleration constants.

Our algorithm the fitness value represents the counter number of point correspond that need to be maximized. The optimization process starts by initializing particles swarm random positions as six parameter (three rotate and three translate) and velocities. Next, for each particle calculate fitness value. This value Compare with thepbest ${ }_{i}$.If new better than current, update pbest ${ }_{i}$. Then, Update the position and velocity of a particle by using Eqs. (13, 14). This process is repeated until the maximum iterations is reached or condition of algorithm is satisfied. Result the global optimal solution of gbest is optimal transformation parameters for the calculation of point cloud data, get the final registration of point cloud.

## 5 Experimental Setup

The algorithm was run on laptop with the following specifications (Processor: Intel(R) Core(TM) i7-4702MQ CPU @ 2.20 GHz 2.19 GHz , installed memory (RAM) 8.00 GB , and System Type: 64-bit Operating system, x64-based
processor with Windows edition Windows 8.1@2013 Microsoft Corporation System). The proposed algorithm was developed using visual Fortran90.

### 5.1 Datasets Descriptions

In order to evaluate the quality of the proposed features and comparison methods, a series of experiments have been conducted using benchmark dataset and real point cloud scans captured by terrestrial laser scanners.

### 5.1.1 Benchmark

The Stanford 3D Scanning Repository was a computer graphics 3D test model such as Stanford bunny and Dragon model. This was scanned by Cyberware 3030 MS scanner.

First model test was Stanford bunny developed by Greg Turk and Marc Levoy in 1994[27]at Stanford University. This mode which consists of 10 scans and total size of scans: 362,272 points. Registration was tested using two from the ten scans, two scans registration were 315 degree scan with the top 3 scan. One of these two scans contains 35158 points (used as Source scene). The other one contains 35921 points (used as moving scene). Both scans in different local coordinate systems were shown in Fig (1a).

Second model test was Dragon which consists of 70 scans and total size of scans: $2,748,318$ points. Two scans registration were dragon Mouth4_0 scan with the dragon Nook2_0 scan. One of these two scans contains 184922 points (used as Source scene). The other one contains 207370 points (used as moving scene). Both scans in different local coordinate systems were shown in Fig (1b).


Fig. 1. Original 3D scan from different datasets. (a) Stanford bunny (b) Dragon.

### 5.1.2 Measured data in lab Real 3D Point Cloud Data from Terrestrial Lidar

Real precise 3d data is nowadays obtained by using professional 3d laser scanners. On the other hand, efficient, largescale 3D point clouds processing is required to process very large amount of data. Terrestrial Light Detection and Ranging is a technology for 3D measurement of object surfaces. The experiments were carried out using real data set were taken from the 3D laser scanner (I-site). The scanning are conducted by the graphics team in the Informatics Research Institute, City of Scientific Research and Technological Applications (SRTA-City), Alexandria, Egypt. All of scans were taken from different positions using the terrestrial laser scanner Imaging-System I-site 4400.
Fig (2a) datasets were collected from the sphinx site in Giza near the city of Cairo. Source scene and moving scene were picked for evaluation consist of 18120 and 41205 raw 3D points respectively. Both scans in different local coordinate system.
Fig (2b) datasets were collected from the Babylon Fortress ('Bawabet Amr) located in old Cairo. All of scans Source scene and moving scene were picked for evaluation consist of 458845 and 674575 raw 3D points respectively.


Fig. 2 represent point cloud with vertex color that was obtained from the RIEGL LMS-Z620 laser scanner of (a) sphinx (b) Bawabet Amr

## 6 Experimental Results

The registration is performed using the particle swarm optimization algorithm (PSO) based on three different descriptors (Distance, Heat kernel and Gaussian curvature) as described in section 2. Experiment goal is estimated to the sixparameter to rigid-body registration of two scenes in different view with robust and effective. To test the robustness and effectiveness of the proposed algorithm, applying in the benchmark datasets and compare the performance of our algorithms with ICP [28].

Table 1: The RMSE of registration with using different methods on Stanford Bunny and Dragon datasets

| Dataset | Method | RMSE |
| :--- | :--- | :---: |
| Stanford | PSO with Distance | 0.001 |
|  | PSO with Heat kernel | 0.163 |
|  | PSO with Gaussian curvatures | 0.220 |
|  | ICP algorithm | 0.914 |
| Bragon | PSO with Distance | 0.002 |
|  | PSO with Heat kernel | 0.210 |
|  | PSO with Gaussian curvatures | 0.533 |
|  | ICP algorithm | 0.30 |

Fig. 3 shows the 3D registration results of Stanford bunny and Dragon. The registration results shown below are based on these datasets. From left to right the figure shows two original scanning ( $a, b$ ) then apply different algorithm to registration. First algorithm depend on distance between point cloud and result show in (c, d), second and third algorithm depend on descriptor (heat kernel and curvature) represent by (e, f, g, h) respectively. Finally, ICP algorithm which can't alignment two cases as shows in (i, j ). The error of each case represents in table1.


Fig.3.result of different algorithm on Stanford bunny and Dragon datasets.
All registration method was also tested on the real dataset to verify its applicability on a more cluttered scene. Fig. 4 shows the 3D registration results of sphinx and Babylon Fortress. The error in this case shows in table2. It can be seen that good performance is achieved on the both scene data by the PSO with Distance registration method.


Fig (4): shows different algorithm applied on different datasets (a, b) two unregistered scans. (c, d)registration after applying our algorithm in distance case. (e, f)Registration after applying our algorithm in Heat kernel case. ( $\mathrm{g}, \mathrm{h}$ ) Registration after applying our algorithm in Gaussian curvatures case. (j, k) Registration using ICP.

Table 2: The RMSE of registration with using different methods on sphinx and Babylon Fortress datasets

| Dataset | Method | RMSE |
| :--- | :--- | :---: |
| Sphinx | PSO with Distance | $\mathbf{0 . 1 4}$ |
|  | PSO with Heat kernel | $\mathbf{0 . 2 8}$ |
|  | PSO with Gaussian curvatures | $\mathbf{0 . 2 0}$ |
|  | ICP algorithm | $\mathbf{0 . 3 9}$ |
| Babylon | PSO with Distance | $\mathbf{0 . 1 4}$ |
|  | PSO with Heat kernel | $\mathbf{0 . 2 1}$ |
|  | PSO with Gaussian curvatures | $\mathbf{0 . 1 5}$ |
|  | ICP algorithm | $\mathbf{0 . 1 9}$ |

Conclusions

In this research, a new registration techniques is presented by using particle swarm with three different descriptors (distance, heat kernel, Gaussian curvature). The experiments are done on branchmark datasets and real data from 3D laser scanner. From the results, the used distance descriptor outperformed the heat kernel and Gaussian curvature descriptor in accuracy. While, Heat kernel and Gaussian curvature descriptor take less time than distance descriptor.

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